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Low-dimensional model of a supersonic rectangular jet

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The proper orthogonal decomposition method is applied to the analysis of particle image velocimetry data obtained for a supersonic rectangular jet operated at underexpanded conditions. Phase-locked velocity field data were used to calculate the eigenfunctions and the eigenvalues. It was found that a large fraction of the total energy is contained within the first two modes. The essential features of the jet are thus captured with only two functions. A low-dimensional model for the dynamical behavior is then constructed using Galerkin projection of the isentropic compressible Navier-Stokes equations. The reduced model compares reasonably well with the experimental findings.

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I. INTRODUCTION

This paper is motivated by our attempt to provide a method of velocity field data analysis aimed at obtaining low-dimensional approximate description of high speed jets. The velocity field data is obtained experimentally using the particle image velocimetry (PIV) technique. Recent developments in the statistical technique of proper orthogonal decomposition (POD) seems to offer some hope to capture the spatial as well as temporal behavior of energetic large-scale structures in a variety of turbulent shear flows. Data analysis using POD is often conducted to extract "mode shapes" or basis functions from experimental data for subsequent use in Galerkin projections that yield low-dimensional models [1]. This enables one to build efficient reduced order models based on the first few dynamically important POD modes, thus serving as potential substitute for computationally intensive simulations.

The POD method was originally suggested by Lumely [2] to extract organized large-scale structures from turbulent flows. The method provides a set of optimized orthonormal basis functions for an ensemble of data. The most important property of POD is its optimality in the sense that it provides the most efficient way of capturing the dominant features of an infinite-dimensional process with only few functions.

Since the introduction of POD as a tool to extract coherent structure in turbulent flows, many studies on the subject have been published. The method has proved to be a powerful tool in educing coherent structures. It has been applied to many kinds of flows such as, boundary layers [3,4], bounded flows [5–7], shear layers [8–10], turbulent jets [11–17], and compressible flows [18,19]. In most of these cases, it has been found that few modes contain a high percentage of the energy. It is important to point out that these flows have been mostly at subsonic velocities and the information is limited due to coarse spatial resolution of the velocity measuring sensors (hot-wire probes). As a result, the velocity field generally yields few modes, which contain most of the energy. However, with direct numerical simulation, it is possible to resolve spatially a great number of scales within the region of interest. But the flows analyzed have been at low Reynolds numbers [4,7,19]. On the other hand, using whole field techniques such as laser induced fluorescence, interferometry tomography, and PIV, it is possible to obtain data with high spatial resolution at high Reynolds numbers. In these cases, the POD method has shown energy distributed among larger number of modes [14–16], since many scales are captured in an instantaneous flow field. Most of the previous investigations have developed low-dimensional order model using Galerkin projection and the flows studied have been in the incompressible regime [3,4,10,18,20]. The only exception is the work of Rowley *et al.* [19], which dealt with compressible flows at subsonic Mach numbers.

In the present study, POD method is applied to PIV data set that was obtained from the phase-locked measurements of synoptic velocity field of an underexpanded rectangular jet. The screeching jet is dominated by coherent large-scale structures generated by the inherent global instability set up by a feedback mechanism [21–23]. The feedback cycle starts with a disturbance in the shear layer that is convected downstream and come in contact with shock cell boundary. This interaction, particularly at the end of the shock cell, produces intense sound wave. This sound wave propagates upstream in the ambient medium, interacts with the incipient shear layer at the nozzle exit and produces a new downstream traveling disturbance that continues the feedback cycle.

It is an attempt in our endeavor in providing a low order dynamic model suitable to predict the main characteristics of supersonic jets. In this paper we will apply the technique to describe the structure of a screeching rectangular jet.

II. THEORETICAL BACKGROUND

A. Brief remarks on the POD method

In the present study, the procedure outlined in Berkooz *et al.* [24] is followed. The idea behind POD is to find a basis function $\{\boldsymbol{\varphi}\}$ in which the ensemble data $\{\mathbf{u}\}$ is optimally represented. In this paper, $\mathbf{u}$ is considered a real random vector of length M with temporal and spatial dependence. Thus, we are looking for a function that maximizes the inner product with $\mathbf{u}$, which is given as

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